LIBERTY PAPER SET

STD. 10 : Mathematics (Basic) [N-018(E)]

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 4**

Section-A

1. (D) No solution **2.** (A) $\sqrt{2}, -\sqrt{2}$ **3.** (C) 10 d **4.** (C) 10 **5.** (A) 1 **6.** (A) 30 **7.** 1 **8.** $-\frac{b}{a}$ **9.** 1 **10.** $\frac{1}{2}$ **11.** Intersection **12.** 50 **13.** False **14.** True **15.** True **16.** True **17.** 12 **18.** 7 cm **19.** $\frac{5}{9}$ **20.** $\frac{8x - 9y}{7}$ **21.** (c) $3\pi r^2$ **22.** (a) $\pi r^2 h$ **23.** (b) $\pi r^2 - \frac{\pi r^2 \theta}{360}$ **24.** (c) $\frac{1}{2}\pi r^2$

Section-B

25. $6x^2 - 7x - 3 = 0$

$\therefore 6x^2 - 9x + 2x - 3 = 0$

$\therefore 3x(2x - 3) + 1(2x - 3) = 0$

$\therefore (2x - 3)(3x + 1) = 0$

$\therefore 2x - 3 = 0$ and $3x + 1 = 0$

$\therefore x = \frac{3}{2}$ and $x = -\frac{1}{3}$

26. Let the quadratis polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$\therefore \alpha\beta = \frac{1}{4}$ and $\alpha + \beta = \frac{1}{4}$

$\therefore \frac{c}{a} = -\frac{1}{4}$ and $-\frac{-b}{a} = \frac{1}{4}$

$\therefore a = 4, b = -1, c = -1$

So, one quadratic polynomial which become the given conditions is $4x^2 - x - 1$. You can check that any other quadratic polynomial that become these condition will be of the form k $(4x^2 - x - 1)$, where k is real.

27. $2x^2 - 5x + 3 = 0$

$\therefore 2x^2 - 2x - 3x + 3 = 0$

$\therefore 2x(x - 1) - 3(x - 1) = 0$

$\therefore (x - 1)(2x - 3) = 0$

$\therefore x - 1 = 0$ or $2x - 3 = 0$

$\therefore x = 1$ $\therefore x = \frac{3}{2}$

$\therefore$ Roots of the equation : $1, \frac{3}{2}$

28. Here, $a = 21$, $d = 18 - 21 = -3$

Suppose, n^{th} term $a_n = -81$

$a_n = a + (n - 1)d$

$\therefore -81 = 21 + (n - 1)(-3)$

$\therefore -81 = 21 - 3n + 3$

$\therefore -81 = 24 - 3n$

$\therefore -27 = 8 - n$

$\therefore n = 8 + 27$

$\therefore n = 35$

Therefore, the 35^{th} term of the given AP is -81

29. Here, $a = 15$, $d = 10 - 15 = -5$ and $n = 10$

$a_n = a + (n - 1)\,d$

$\therefore\ a_{10} = 15 + (10 - 1)(-5)$

$\therefore\ a_{10} = 15 - 45$

$\therefore\ a_{10} = -30$

Therefore, the 10^{th} term of the given AP is – 30.

30. $AB = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(1-5)^2 + (2-8)^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$= 2\sqrt{13}$

Thus, the distance between the given points is $2\sqrt{13}$.

31. Let, the given points be P(1, 5), Q(2, 3) & R(– 2, – 11).

$PQ = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{1+4} = \sqrt{5} = 2.24$

$QR = \sqrt{(2+2)^2 + (3+11)^2} = \sqrt{16+196} = \sqrt{212} = 14.56$

$PR = \sqrt{(1+2)^2 + (5+11)^2} = \sqrt{9+256} = \sqrt{265} = 16.28$

$\therefore\ 2.24 + 14.56 = 16.80 \neq 16.28$

$\therefore\ PQ + QR \neq PR$

Same as, $QR + PR \neq PQ$ & $PQ + PR \neq QR$

Therefore, the given points are not collinear.

32. $\sin(A - B) = \frac{1}{2}$ $\qquad \cos(A + B) = \frac{1}{2}$

$\therefore\ \sin(A - B) = \sin 30°$ $\quad \therefore\ \cos(A + B) = \cos 60°$

$\therefore\ A - B = 30°$ ....(1) $\quad \therefore\ A + B = 60°$ ....(2)

Adding equation (1) and (2),

$(A - B) + (A + B) = 30° + 60°$

$\therefore\ A - B + A + B = 90°$

$\therefore\ 2A = 90°$

$\therefore\ A = 45°$

Put $A = 45°$ in equation (1),

$A - B = 30°$

$\therefore\ B = A - 30°$

$\therefore\ B = 45° - 30°$

$\therefore\ B = 15°$

Hence, $A = 45°$ and $B = 15°$.

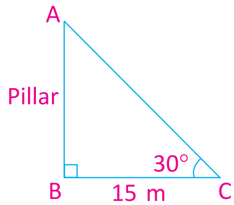
33. ***sin* 60° *cos* 30° + *sin* 30° *cos* 60°**

$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

34.



Here in Δ ABC, ∠B = 90°

$\therefore \quad \tan 30° = \frac{AB}{BC}$

$\therefore \quad \frac{1}{\sqrt{3}} = \frac{AB}{15}$

$\therefore \quad AB = \frac{15}{\sqrt{3}}$

$\therefore \quad AB = 5\sqrt{3}$

Therefore the height of the pillar is $5\sqrt{3}$ m.

35. Volume of cube = x^3

$\therefore\ 125 = x^3$

$\therefore\ 5^3 = x^3$

$\therefore\ x = 5$ cm

$\therefore\ l = 2x = 2(5) = 10$ cm

$b = x = 5$ cm

$h = x = 5$ cm

CSA of cuboid = $2\ (lb + bh + hl)$

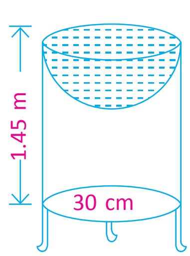
$= 2\ [(10 \times 5) + (5 \times 5) + (10 \times 5)]$

$= 2\ [50 + 25 + 50]$

$= 2\ (125)$

$= 250$ cm^2

36.



Cylinder	Hemisphere
$h = 1.45$ m $= 145$ cm	$r = 30$ cm
$r = 30$ cm	

The total surface area of the bird-drinker.

= CSA of cylinder + CSA of hemisphere

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\ (h + r)$

$= 2 \times \frac{22}{7} \times 30 \times (145 + 30)$

$= 2 \times \frac{22}{7} \times 30 \times 175$

$= 33000\ cm^2$

Thus, TSA of the bird-bath is $33000\ cm^2 = 3.3\ m^2$

37. mean $\bar{x} = a + \frac{\sum fiui}{\sum fi} \times h$

$= 475 + \frac{29}{30} \times 15$

$= 475 + 14.5$

$= 489.5$

38. Let us assume that,

Jayshree's present age = x

Purvi's present age = y

Five years ago,

Jayshree = $x - 5$

Purvi = $y - 5$

$\therefore$ As per condition $(x - 5) = 3\ (y - 5)$

$\therefore\ x - 5 = 3y - 15$

$\therefore\ x - 3y = -15 + 5$

$\therefore\ x - 3y = -10$ ...(1)

10 years from now,

Jayshree will be $x + 10$ & Purvi will be $y + 10$

As per condition $(x + 10) = 2\ (y + 10)$

$\therefore\ x + 10 = 2y + 20$

$\therefore\ x - 2y + 20 - 10$

$\therefore\ x - 2y = 10$ ...(2)

Subtract (2) from (1),

$x - 3y = -10$

$x - 2y = 10$

$- \quad + \qquad -$

$\therefore\ -y = -20$

Put y = 20 in eqn (1),

$x - 3y = -10$

$\therefore\ x - 3(20) = -10$

$\therefore\ x - 60 = -10$

$\therefore\ x = -10 + 60$

$\therefore\ x = 50$

Jayshree's present age = 50 years, Purvi's present age = 20 years.

39. $3x - 5y - 4 = 0$ ...(1)

$\therefore x = \dfrac{5y+4}{3}$ ...(2)

$9x = 2y + 7$

$\therefore 9x - 2y - 7 = 0$ ...(3)

Put equation (2) in equation (3),

$9x - 2y - 7 = 0$

$\therefore 9\left(\dfrac{5y+4}{3}\right) - 2y - 7 = 0$

$\therefore 15y + 12 - 2y - 7 = 0$

$\therefore 15y - 2y = -12 + 7$

$\therefore 13y = -5$

$\therefore y = \dfrac{-5}{13}$

Put $y = \dfrac{-5}{13}$ equation (2)

$x = \dfrac{5y+4}{3}$

$\therefore x = \dfrac{5\left(\frac{-5}{13}\right)+4}{3} = \dfrac{\frac{-25}{13}+4}{3} = \dfrac{-25+52}{39} = \dfrac{27}{39} = \dfrac{9}{13}$

$\therefore x = \dfrac{9}{13}$

The solution of the equation : $x = \dfrac{9}{13}$, $y = \dfrac{-5}{13}$

40. Here, $a_2 = a + d = 14$ and $a_3 = a + 2d = 18$.

$\therefore a + d = 14$

$a + 2d = 18$

$-\quad -\quad -$

$\therefore -d = -4$

$\therefore d = 4$

Put $d = 4$ in $a + d = 14$

$a + d = 14$

$\therefore a + 4 = 14$

$\therefore a = 14 - 4$

$\therefore a = 10$

Now, $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$\therefore S_{51} = \dfrac{51}{2}[2(10) + (51-1)4]$

$= \dfrac{51}{2}[20 + 200]$

$= \dfrac{51}{2} \times 220$

$= 51 \times 110$

$\therefore S_{51} = 5610$

41. Suppose, the point P $(x, 0)$ on the X-axis divides the line segment connecting the points A $(1, -5)$ and B $(-4, 5)$in the ratio $m_1 : m_2$.

The co-ordinates of the dividing point P = $\left(\dfrac{m_1x_2 + m_2x_1}{m_1 + m_2}, \dfrac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$

$$(x, 0) = \left(\frac{-4m_1 + m_2}{m_1 + m_2}, \frac{5m_1 - 5m_2}{m_1 + m_2}\right)$$

$$0 = \frac{5m_1 - 5m_2}{m_1 + m_2} \quad \text{(comparing } y \text{ co-ordinates)}$$

$\therefore\ 0 = 5m_1 - 5m_2$

$\therefore\ 5m_1 = 5m_2$

$\therefore\ m_1 = m_2$

$\therefore\ \dfrac{m_1}{m_2} = \dfrac{1}{1}$

$\therefore\ m_1 : m_2 = 1 : 1$

$$x = \frac{-4m_1 + m_2}{m_1 + m_2} \quad \text{(comparing } x \text{ co-ordinates)}$$

$\therefore\ x = \dfrac{-4(1)+1}{1+1}$

$\therefore\ x = \dfrac{-4+1}{2}$

$\therefore\ x = -\dfrac{3}{2}$

Thus, the x-axis divides the line segment connecting the points A $(1, -5)$ and B $(-4, 5)$ at point $\left(-\dfrac{3}{2}, 0\right)$ in a 1 : 1 ratio.

42. Here, Q $(0, 1)$ is equidistant from P $(5, -3)$ and R $(x, 6)$ as given :

$\therefore$ PQ = QR

$\therefore$ $PQ^2 = QR^2$

$\therefore\ (5-0)^2 + (-3-1)^2 = (0-x)^2 + (1-6)^2$

$\therefore\ 25 + 16 = x^2 + 25$

$\therefore\ x^2 = 16$

$\therefore\ x = \pm 4$

If, $x = 4$;

$QR = \sqrt{(0-x)^2+(1-6)^2}$, $\qquad PR = \sqrt{(5-x)^2+(-3-6)^2}$

$\therefore\ QR = \sqrt{(0-4)^2+(1-6)^2}$ $\qquad \therefore\ PR = \sqrt{(5-4)^2+(-3-6)^2}$

$\therefore\ QR = \sqrt{16+25}$ $\qquad \therefore\ PR = \sqrt{1+81}$

$\therefore\ QR = \sqrt{41}$ $\qquad \therefore\ PR = \sqrt{82}$

Same as, $x = -4$;

$QR = \sqrt{(0-x)^2+(1-6)^2}$ $\qquad PR = \sqrt{(5-x)^2+(-3-6)^2}$

$\therefore\ QR = \sqrt{(0+4)^2+(-5)^2}$ $\qquad \therefore\ PR = \sqrt{(5+4)^2+(-3-6)^2}$

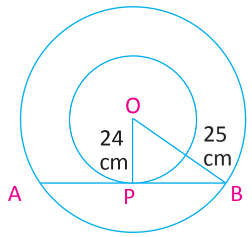
$\therefore\ QR = \sqrt{16+25}$ $\qquad \therefore\ PR = \sqrt{81+81}$

$\therefore\ QR = \sqrt{41}$ $\qquad \therefore\ PR = \sqrt{162}$

$\therefore\ PR = \sqrt{81 \times 2}$

$\therefore\ PR = 9\sqrt{2}$

43.



Here, We have,

$r_1 = 25$ cm and $r_2 = 24$ cm

Length of the chord $= 2\sqrt{r_1^2 - r_2^2}$

$= 2\sqrt{(25)^2 - (24)^2}$

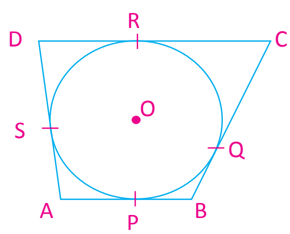
$= 2\sqrt{625 - 576}$

$= 2\sqrt{49}$

$= 2\ (7)$

$= 14$ cm

44.



Let the sides AB, BC, CD and DA of the quadrilateral ABCD touch the O centric circle at points P, Q, R and S respectively.

$\therefore$ AP = AS ...(1)

BP = BQ ...(2)

CR = CQ ...(3)

DR = DS ...(4)

Add equation (1), (2), (3) and (4)

AP + BP + CR + DR = AS + BQ + CQ + DS

$\therefore$ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

$\therefore$ AB + CD = AD + BC

45. Here we get the information as shown in the table below using $a = 225$ and $h = 50$ to use the deviation method.

Daily expenditure (in ₹)	(f_i)	x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
100 – 150	4	125	– 2	– 8
150 – 200	5	175	– 1	– 5
200 – 250	12	225 = a	0	0
250 – 300	2	275	1	2
300 – 350	2	325	2	4
Total	$\Sigma f_i = 25$	–	–	$\Sigma f_i u_i = -7$

Mean $\bar{x} = a + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

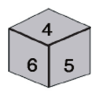
$\therefore \bar{x} = 225 + \frac{-7}{25} \times 50$

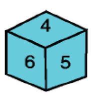
$\therefore \bar{x} = 225 - 14$

$\bar{x} = 211$

So, mean daliy expenditure on food is ₹ 211.

46.





	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

The number of possible outcomes = 6 × 6 = 36

(i) The outcomes favourable to the event E 'the sum of the two numbers is 8' denoted by E = 5[(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)]

Hence, $P(E) = \frac{5}{36}$

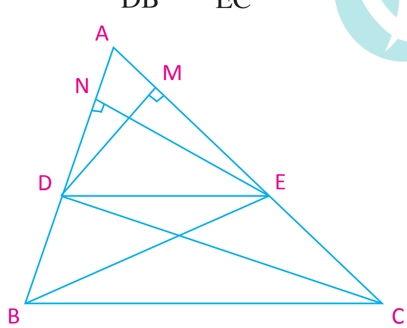
(ii) All the outcomes are favourable to the event G, 'sum of two numbers $\leq$ 12'.

So, $P(G) = \frac{36}{36} = 1$

47. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

Given: In ABC, a line parallel to side BC intersects AB and AC at D and E respectively.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$



Proof: Join BE and CD and also draw DM ⊥ AC and EN ⊥ AB.

Then, $ADE = \frac{1}{2} \times AD \times EN$,

$BDE = \frac{1}{2} \times DB \times EN$,

$ADE = \frac{1}{2} \times AE \times DM$ and

$DEC = \frac{1}{2} \times EC \times DM$.

$$\therefore \frac{ADE}{BDE} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \qquad ...(1)$$

$$\text{and } \frac{ADE}{DEC} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad ...(2)$$

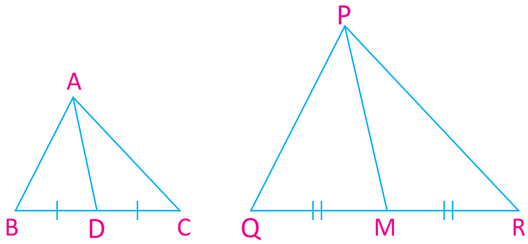
Now, Δ BDE and Δ DEC are triangles on the same base DE and between the parallel BC and DE.

then, BDE = DEC ...(3)

Hence from eq[n]. (1), (2) and (3),

$$\frac{AD}{DB} = \frac{AE}{EC}$$

48.



AD and PM are the medians of Δ ABC and Δ PQR respectively.

∴ BC = 2 BD and QR = 2 QM

Now, Δ ABC ~ Δ PQR

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\therefore \frac{AB}{PQ} = \frac{2\,BD}{2\,QM}$$

$$\therefore \frac{AB}{PQ} = \frac{BD}{QM}$$

Also, ∠ABC = ∠PQR

∴ ∠ABD = ∠PQM

Now, Δ ABD and Δ PQM,

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ and } \angle ABD = \angle PQM$$

∴ Δ ABD ~ Δ PQM (SAS criterion)

$$\therefore \frac{AB}{PQ} = \frac{AD}{PM}$$

49. We have,

$$x - \frac{1}{x} = 3$$

$$\frac{x^2 - 1}{x} = 3$$

$x^2 - 1 = 3x$

$x^2 - 3x - 1 = 0$

Let's compare with $ax^2 + bx + c = 0$

a = 1, b = –3, c = –1

$b^2 - 4ac = (-3)^2 - 4\,(1)\,(-1) = 9 + 4 = 13$

$$x = \frac{-b + 4al}{2a}$$

$$= \frac{-(-3) \pm \sqrt{13}}{2(1)}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

Therefore, the roots of eq^n : $\frac{3+\sqrt{13}}{2}$, $\frac{3-\sqrt{13}}{2}$

50. Here, $a = 21$, $d = 18 - 21 = -3$

Suppose, n^{th} term $a_n = -81$

$a_n = a + (n - 1)\, d$

$\therefore\ -81 = 21 + (n - 1)(-3)$

$\therefore\ -81 = 21 - 3n + 3$

$\therefore\ -81 = 24 - 3n$

$\therefore\ -27 = 8 - n$

$\therefore\ n = 8 + 27$

$\therefore\ n = 35$

Therefore, the 35^{th} term of the given AP is -81

Now, Suppose n^{th} term $a_n = 0$

$a_n = a + (n - 1)\, d$

$\therefore\ 0 = 21 + (n - 1)(-3)$

$\therefore\ 0 = 21 - 3n + 3$

$\therefore\ 3n = 21 + 3$

$\therefore\ 3n = 24$

$\therefore\ n = 8$

So, the eighth term is 0.

51.

Daily pocket allowance (₹)	Number of childern (f_i)	x_i	u_i	$f_i u_i$
11 – 13	7	12	–4	–28
13 – 15	6	14	–3	–18
15 – 17	9	16	–2	–18
17 – 19	13	18	–1	–13
19 – 21	f	$20 = a$	0	0
21 – 23	5	22	1	5
23 – 25	4	24	2	8
Total	$44 + f$	–	–	–64

Mean $\overline{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) h$

$\therefore\ 18 = 20 + \left(\frac{-64}{44+f}\right) 2$

$\therefore\ 18 - 20 = \frac{-128}{44+f}$

$\therefore\ -2 = \frac{-128}{44+f}$

$\therefore\ 44 + f = \frac{-128}{-2}$

$\therefore\ 44 + f = 64$

$\therefore\ f = 64 - 44$

$\therefore\ f = 20$

Hence, the missing frequcncy f is 20.

52.

class	frequency (f_i)	cf
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$25 + x$
30 – 40	15	$40 + x$
40 – 50	y	$40 + x + y$
50 – 60	5	$45 + x + y$

Here, M = 28.5

$n = 60$

Median class = 20 – 30

l = lower limit of median class = 20

n = total frequency = 60

cf = cumulative frequency of class preceding the median class = $5 + x$

f = frequency of median class = 20

h = class size = 10

$$M = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\therefore\ 28.5 = 20 + \left(\frac{\frac{60}{2} - (5 + x)}{20}\right) \times 10$$

$$\therefore\ 28.5 - 20 = \frac{(30 - 5 - x) \times 10}{20}$$

$$\therefore\ \frac{8.5 \times 20}{10} = 25 - x$$

$\therefore\ 17 = 25 - x$

$x = 25 - 17$

$x = 8$

Now, $\Sigma f_i = n = 60$

$\therefore 45 + x + y = 60$

$\therefore 45 + 8 + y = 60$

$\therefore 53 + y = 60$

$\therefore y = 60 - 53$

$\therefore y = 7$

Thus, $x = 8$ and $y = 7$.

53. Total number of balls = 3 + 5 + 7 = 15

(i) Suppose event A is getting a red ball

$\therefore P(A) = \frac{\text{Number of red ball}}{\text{Total number of ball}}$

$\therefore P(A) = \frac{3}{15} = \frac{1}{5}$

(ii) Suppose event B is getting a black ball

$\therefore P(B) = \frac{\text{Number of black ball}}{\text{Total number of ball}}$

$\therefore P(B) = \frac{5}{15} = \frac{1}{3}$

(iii) Suppose event C is getting a white ball

$\therefore P(C) = \frac{\text{Number of black ball}}{\text{Total number of ball}}$

$\therefore P(C) = \frac{7}{15}$

(iv) Suppose event D is complementary of event C.

$\therefore P(D) = 1 - P(C)$

$\therefore P(D) = 1 - \frac{7}{15} = \frac{8}{15}$

54. Here, the number of possible outcomes is 6.

(i) Suppose event A is greater than 4. (5 and 6)

$\therefore P(A) = \frac{\text{Number of greater than 4}}{\text{Number of possible outcomes}}$

$\therefore P(A) = \frac{2}{6} = \frac{1}{3}$

(ii) Suppose event B is less than 4 or 4.

Here event B is complementary event of event A.

$\therefore P(B) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$

(iii) Suppose event C is greater than 3 and less than 5 (only 4)

Here event B is complementary event of event A.

$$\therefore P(C) = \frac{\text{Number of greater than 3 and less than 5}}{\text{Number of possible outcomes}}$$

$$\therefore P(C) = \frac{1}{6}$$

(iv) Suppose event D is an even number.

(2, 4, 6 = 3)

$$\therefore P(D) = \frac{\text{Number of an even number}}{\text{Number of possible outcomes}}$$

$$\therefore P(D) = \frac{3}{6} = \frac{1}{2}$$